Period 3, Sept 25, 2024

$$y = (3x^2+2x)^2 = (3x^2+2x)(3x^2+2x) = 9x^4+6x^3+6x^3+4x^2$$

$$y = 9x^4+12x^3+4x^2$$

$$\frac{dy}{dx} = 36x^3+36x^2+8x$$

$$y=(3x^2+2x)^2 \Rightarrow y=u^2$$

$u = 3x^2+2x$ $\qquad \frac{dy}{du} = 2u$

$$\frac{du}{dx} = 6x+2$$

$$\frac{\cancel{du}}{dx}\cdot\frac{dy}{\cancel{du}} = (6x+2)(2u) = (6x+2)\cdot 2(3x^2+2x)$$

$$(12x+4)(3x^2+2x)$$

$$36x^3+24x^2+12x^2+8x$$

$$36x^2+36x^2+8x$$

$$y = (3x^2+2x)^{157} \Rightarrow y=u^{157}$$

$$\boxed{\frac{dy}{dx} = 157(6x+2)(3x^2+2x)^{156}}$$

$u = 3x^2+2x$ $\qquad \frac{dy}{du} = 157\cdot u^{156}$

$$\frac{du}{dx} = 6x+2$$

$$\frac{\cancel{du}}{dx}\cdot\frac{dy}{\cancel{du}} = \frac{dy}{dx} = (6x+2)(157u^{156}) = 157(6x+2)(3x^2+2x)^{156}$$

$$y = \sin(3x^2+2x) \Rightarrow y = \sin u$$

$u = 3x^2+2x$ $\qquad \frac{dy}{du} = \cos u$

$$\frac{du}{dx} = 6x+2$$

$$\frac{\cancel{du}}{dx}\cdot\frac{dy}{\cancel{du}} = (6x+2)\cos u$$

$$\frac{dy}{dx} = (6x+2)\cos(3x^2+2x)$$

$$y = \sin^5(3x^2+2x) \Rightarrow y = \sin^5 u = (\sin u)^5 \Rightarrow y = L^5$$

$u = 3x^2+2x$ $\quad L = \sin u$ $\quad \frac{dy}{dL} = 5L^4$

$\frac{du}{dx} = 6x+2$ $\quad \frac{dL}{du} = \cos u$

$$\frac{du}{\cancel{dx}} \cdot \frac{\cancel{dL}}{\cancel{du}} \cdot \frac{dy}{\cancel{dL}} = \frac{dy}{dx}$$

$$(6x+2)(\cos u)(5L^4) = 5(6x+2)\cos(3x^2+2x) \cdot \sin^4 u$$

$$\frac{dy}{dx} = 5(6x+2)\cos(3x^2+2x) \cdot \sin^4(3x^2+2x)$$

$$y = \sin x \cdot \cot^4\left(\sqrt{3x^2+2x}\right)$$

$$\frac{dy}{dx} = \cos x\left(\cot^4(\sqrt{3x^2+2x}\right) + \sin x \frac{d}{dx}\left[\cot^4\sqrt{3x^2+2x}\right]$$

$$y = \cot^4\sqrt{3x^2+2x} \Rightarrow y = \cot^4\sqrt{u}$$

$u = 3x^2+2x$ $\quad L = \sqrt{u} = u^{\frac{1}{2}}$

$\frac{du}{dx} = 6x+2$ $\quad \frac{dL}{du} = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}}$

$$\frac{\cancel{du}}{dx} \cdot \frac{\cancel{dL}}{\cancel{du}} \cdot \frac{\cancel{dn}}{\cancel{dL}} \cdot \frac{dy}{\cancel{dn}}$$

$y = \cot^4 L$

$n = \cot L$

$\frac{dn}{dL} = -\csc^2 L$

$y = n^4$

$\frac{dy}{dn} = 4n^3$

$$\boxed{(6x+2)\left(\frac{1}{2\sqrt{3x^2+2x}}\right)\left(-\csc^2\sqrt{3x^2+2x}\right)\left(4\cot^3\sqrt{3x^2+2x}\right)}$$

$$\frac{dy}{dx} = \cos x\left(\cot^4\sqrt{3x^2+2x}\right) + \sin x\left[\frac{(6x+2)}{2\sqrt{3x^2+2x}} \cdot -\csc^2\sqrt{3x^2+2x} \cdot 4\cot^3\sqrt{3x^2+2x}\right]$$

$$\frac{dy}{dx} = \cos x(\cot^4\sqrt{3x^2+2x}) + \sin x\left[\frac{6x+2}{2\sqrt{3x^2+2x}}\left(-\csc^2\sqrt{3x^2+2x}\right)\left(4\cot^3\sqrt{3x^2+2x}\right)\right.$$

$$h(x) = \frac{F(x)}{g(x)} \Rightarrow h'(x) = \frac{F'(x)\cdot g(x) - F(x)\cdot g'(x)}{(g(x))^2}$$

$$y = 3^2 = 9$$

$$\frac{dy}{dx} = 0$$

37. $f(x) = \dfrac{x^2+c^2}{x^2-c^2}$, c is a constant

$$\cancel{2x^3} - 2xc^2 - \cancel{2x^3} - 2xc^2$$

$$F'(x) = \frac{2x(x^2-c^2) - (x^2+c^2)(2x)}{[(x-c)(x+c)]^2} = \frac{-4xc^2}{(x-c)^2(x+c)^2}$$

38. $f(x) = \dfrac{c^2-x^2}{c^2+x^2}$, c is a constant

$$F'(x) = \frac{-2x(c^2+x^2) - (c^2-x^2)(2x)}{(c^2+x^2)^2} = \frac{-2xc^2 - \cancel{2x^3} - 2xc^2 + \cancel{2x^3}}{(c^2+x^2)^2}$$

$$\frac{-4xc^2}{(c^2+x^2)^2}$$

In Exercises 59–62, evaluate the derivative of the function at the given point. Use a graphing utility to verify your result.

Function	Point
59. $y = \dfrac{1+\csc x}{1-\csc x}$	$\left(\dfrac{\pi}{6}, -3\right)$
60. $f(x) = \tan x \cot x$	$(1, 1)$
61. $h(t) = \dfrac{\sec t}{t}$	$\left(\pi, -\dfrac{1}{\pi}\right)$
62. $f(x) = \sin x(\sin x + \cos x)$	$\left(\dfrac{\pi}{4}, 1\right)$

$$\Rightarrow \frac{dy}{dx} = \frac{-\csc x\cot x(1-\csc x) - (1+\csc x)(+(+\csc x\cot x)}{(1-\csc x)^2}$$

$$\frac{dy}{dx} = \frac{-\csc x\cot x + \cancel{\csc^2 x\cot} x - \csc x\cot x - \cancel{\csc^2 x\cot x}}{(1-\csc^2 x)^2}$$

$$\frac{dy}{dx} = \frac{-2\csc x\cot x}{(1-\csc x)^2}$$

$$\csc\frac{\pi}{6} = 2$$

$$\cot\frac{\pi}{6} = \sqrt{3}$$

$$\frac{-2\cdot 2\cdot\sqrt{3}}{(1-2)^2} = \frac{-4\sqrt{3}}{1} = -4\sqrt{3}$$

$$F(x) = \tan x\cot x = \frac{\cancel{\sin x}}{\cancel{\cos x}}\cdot\frac{\cancel{\cos x}}{\cancel{\sin x}} = 1$$

$$F'(x) = 0$$

$$F(x) = \tan x\cot x$$

$$F'(x) = \sec^2 x\cdot\cot x + \tan x\cdot -\csc^2 x = \frac{1}{\cos^2 x}\cdot\frac{\cancel{\cos x}}{\sin x} - \frac{\cancel{\sin x}}{\cos x}\cdot\frac{1}{\sin^2 x} = \frac{1}{\cos x\sin x} - \frac{1}{\cos x\sin x}$$

$$0$$

61. $h(T)=\frac{\sec T}{T} \Rightarrow h'(T)=\frac{\sec T\tan T\cdot T-\sec T\cdot 1}{T^2}$

$\tan\pi=0$

$\sec\pi=-1$

$h'(\pi)=\frac{-1\cdot 0\cdot\pi-(-1)\cdot 1}{\pi^2}=\frac{1}{\pi^2}$

$F(x)=\sin x(\sin x+\cos x)$ $\quad F'\left(\frac{\pi}{4}\right)$ $\quad \cos\frac{\pi}{4}=\frac{\sqrt2}{2}$ $\quad \sin\frac{\pi}{4}=\frac{\sqrt2}{2}$

$F'(x)=\cos x(\sin x+\cos x)+\sin x(\cos x+-\sin x)$

$F'(x)=\cos\frac{\pi}{4}\left(\sin\frac{\pi}{4}+\cos\frac{\pi}{4}\right)+\cancel{\sin\frac{\pi}{4}\left(\cos\frac{\pi}{4}-\sin\frac{\pi}{4}\right)}=\frac{\sqrt2}{2}\left(\frac{\sqrt2}{2}+\frac{\sqrt2}{2}\right)=\frac{\sqrt2}{2}\cdot\frac{2\sqrt2}{2}$

$\frac{\sqrt2}{2}-\frac{\sqrt2}{2}=0$

$=\frac{2}{2}=1$

45. $g(t)=\sqrt[4]{t}+6\csc t$

$g(T)=T^{\frac14}+6\csc T$

$g'(T)=\frac14 T^{\frac14-1}+6(-\csc T\cot T)$

$g'(T)=\frac{1}{4\sqrt[4]{T^3}}-6\csc T\cot T$

9. $h(x)=\frac{\sqrt{x}}{x^3+1}$

$y=\sqrt{x}$

$\frac{dy}{dx}=\frac{1}{2\sqrt{x}}$

$h'(x)=\frac{\frac{1}{2\sqrt x}(x^3+1)-\sqrt x(3x^2)}{(x^3+1)^2}$

$h'(x)=\frac{\frac{x^3}{2\sqrt x}+\frac{1}{2\sqrt x}-\frac{3x^{2+\frac12}2\sqrt x}{1\,2\sqrt x}}{(x^3+1)^2}$

$h'(x)=\frac{\frac{x^3}{2\sqrt x}+\frac{1}{2\sqrt x}-\frac{6x^{2+\frac12+\frac12}}{2\sqrt x}}{(x^3+1)^2}$

$\frac{\frac{x^3+1-6x^3}{2\sqrt x}}{\frac{(x^3+1)^2}{1}}$

$\frac{-5x^3+1}{2\sqrt x}\cdot\frac{1}{(x^3+1)^2}=\frac{-5x^3+1}{2\sqrt x(x^3+1)^2}$

$y-y_1=m(x-x_1)$

66. $f(x)=\frac{(x-1)}{(x+1)}$, $\left(2,\frac13\right)$

$F'(x)=\frac{1(x+1)-(x-1)(1)}{(x+1)^2}$

$F'(x)=\frac{\cancel{x}+1-\cancel{x}+1}{(x+1)^2}=\frac{2}{(x+1)^2}$

$F'(2)=\frac{2}{(2+1)^2}=\frac{2}{9}$

$y-\frac13=\frac29(x-2)\Rightarrow y=\frac29(x-2)-\frac{1\cdot3}{3\cdot3}$

$y=\frac29x-\frac49-\frac39=\frac29x-\frac79$

46. $h(x) = \frac{1}{x} - 12\sec x$

$h(x) = x^{-1} - 12\sec x$

$h'(x) = -1x^{-1-1} - 12(\sec x \tan x)$

$\frac{-1}{x^2} - 12\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}$

$\frac{-1}{x^2} - \frac{12\sin x}{\cos^2 x}$

68. $f(x) = \sec x, \quad \left(\frac{\pi}{3}, 2\right)$

$F'(x) = \sec x \tan x$

$\sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$

$\tan\frac{\pi}{3} = \frac{\sin\pi/3}{\cos\pi/3} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

$F'\left(\frac{\pi}{3}\right) = 2\cdot\sqrt{3} = m$

$y - 2 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$

64. $f(x) = (x+3)(x^2-2), \quad (-2, 2)$

$F'(x) = 1(x^2-2) + (x+3)(2x)$

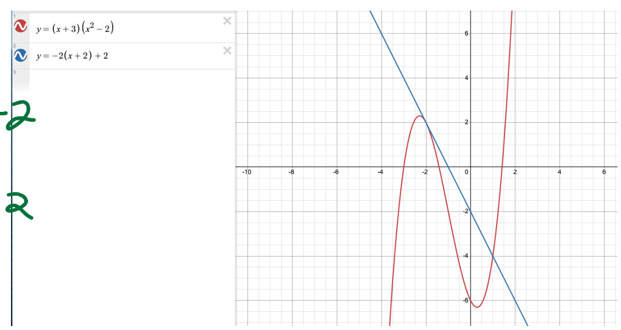
$F'(x) = x^2 - 2 + 2x^2 + 6x = 3x^2 + 6x - 2$

$F'(-2) = 3(-2)^2 + 6(-2) - 12 = \cancel{12}\cancel{-12} - 2$

$m = -2$

$y - 2 = -2(x - (-2))$

$y = -2(x+2) + 2$



33. $f(x) = \frac{2 - \frac{1}{x}}{x - 3}$

$F(x) = \frac{\frac{2x}{x} - \frac{1}{x}}{x-3} = \frac{\frac{2x-1}{x}}{\frac{x-3}{1}} = \frac{2x-1}{x}\cdot\frac{1}{x-3} = \frac{2x-1}{x^2-3x}$

$F(x) = \frac{2x-1}{x^2-3x}$ $\quad F'(x) = \frac{2(x^2-3x) - (2x-1)(2x-3)}{(x-3)^2}$

$F'(x) = \frac{2x^2 - \cancel{6x} - 4x^2 + \cancel{6x} + 2x - 3}{(x-3)^2} = \frac{-2x^2+2x-3}{(x^2-3x)^2}$

27. $f(x) = x\left(1 - \frac{4}{x+3}\right)$

$F(x) = x\left(\frac{x+3}{x+3} - \frac{4}{x+3}\right)$

$F(x) = x\left(\frac{x-1}{x+3}\right) = \frac{x^2-x}{x+3}$

$F'(x) = \frac{(2x-1)(x+3) - (x^2-x)(1)}{(x+3)^2}$

$F'(x) = \frac{2x^2+6x-x-3-x^2+x}{(x+3)^2}$

$F'(x) = \frac{x^2+6x-3}{(x+3)^2}$

28. $f(x) = x^4\left(1 - \frac{2}{x+1}\right)$

$$F(x) = x^4\left(\frac{x+1}{x+1} - \frac{2}{x+1}\right)$$

$$x^4\left(\frac{x+1-2}{x+1}\right)$$

$$F(x) = x^4\left(\frac{x-1}{x+1}\right)$$

$$F(x) = \frac{x^5 - x^4}{x+1}$$

$$\frac{(x+3)\,x}{(x+3)\,1} - \frac{4x}{x+3} = \frac{x^2+3x}{x+3} - \frac{4x}{x+3}$$

$$F'(x) = \frac{(5x^4 - 4x^3)(x+1) - (x^5 - x^4)(1)}{(x+1)^2}$$

$$F'(x) = \frac{5x^5 - 4x^4 + 5x^4 - 4x^3 - x^5 + x^4}{(x+1)^2}$$

$$F'(x) = \frac{4x^5 + 2x^4 - 4x^3}{(x+1)^2} = \frac{2x^3(2x^2+x-2)}{(x+1)^2}$$

47. $y = \dfrac{3(1 - \sin x)}{2\cos x}$

48. $y = \dfrac{\sec x}{x}$

49. $y = -\csc x - \sin x$

50. $y = x\sin x + \cos x$

47. $\dfrac{3 - 3\sin x}{2\cos x} = y$

$$\frac{dy}{dx} = \frac{-3(\cos x)(2\cos x) - (3 - 3\sin x)(-2\sin x)}{(2\cos x)^2} = \frac{-6\cos^2 x + 6\sin x - 6\sin^2 x}{(2\cos x)^2}$$

$$\frac{-6(\cos^2 x + \sin^2 x - \sin x)}{(4\cos^2 x)}$$

$$\frac{-3(\cancel{1-\sin x})}{2(1+\sin x)\cancel{(1-\sin x)}} = \frac{-6(1-\sin x)}{2(1-\sin^2 x)} = \frac{-6(1-\sin x)}{4\cos^2 x}$$

48. $y = \frac{\sec x}{x}$

$$\frac{dy}{dx} = \frac{\sec x \tan x \cdot x - \sec x \cdot 1}{x^2}$$

$$\frac{dy}{dx} = \frac{\sec x(x\tan x - 1)}{x^2}$$

49. $y = -\csc x - \sin x$

$$\Rightarrow \frac{dy}{dx} = -(-\csc x \cot x) - \cos x$$

$$= \csc x \cot x - \cos x$$

$$\frac{\cos^3 x}{\cos^2 x} = \frac{\cos x - \cos^3 x}{\sin^2 x} = \frac{\cos x(1-\sin^2 x)}{\sin^2 x} = \frac{\cos x}{\sin^2 x} - \frac{\cos x \cdot \sin^2 x}{\sin^2 x}$$

$$\boxed{\cos x \cdot \tan^2 x}$$

$$4x \neq \frac{x^2+3x}{x} = \frac{\cancel{x}(x+3)}{\cancel{x}}$$

$$x = 14$$

$$\frac{x+5}{x} \neq 5$$

$$\frac{14+5}{14} = \frac{19}{14} \neq 5$$

50. $y = x\sin x + \cos x$

$$\frac{dy}{dx} = \cancel{1 \cdot \sin x} + x \cdot \cos x + \cancel{-\sin x} = x\cos x$$

$$f(x) = \sqrt[3]{\left(x^2 - 1\right)^2} \Rightarrow y = \sqrt[3]{u^2} = u^{2/3}$$

$$u = x^2 - 1$$

$$\frac{du}{dx} = 2x$$

$$\frac{dy}{du} = \frac{2}{3}u^{\frac{2}{3}-1} = \frac{2}{3}u^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{u}}$$

$$\frac{\cancel{du}}{dx}\cdot\frac{dy}{\cancel{du}} = 2x\left(\frac{2}{3\sqrt[3]{u}}\right) = \frac{4x}{3\sqrt[3]{x^2-1}}$$

: $$g(t) = \frac{-7}{(2t-3)^2}$$

$$y = \frac{-7}{(2T-3)^2} \Rightarrow y = \frac{-7}{u^2} = -7u^{-2}$$

$$u = 2T - 3$$

$$\frac{du}{dT} = 2$$

$$\frac{dy}{du} = 14u^{-3}$$

$$\frac{\cancel{du}}{dt}\cdot\frac{dy}{\cancel{du}} = 2(14u^{-3}) = \frac{28}{u^3}$$

$$\frac{dy}{dT} = \frac{28}{(2T-3)^3}$$

$$y = \sin(5x^3 - 4x)$$

$$\frac{dy}{dx} = (15x^2 - 4)(\cos(5x^3 - 4x))$$

$$y = e^{5x^4 - 3x^2}$$

$$u = 5x^4 - 3x^2$$

$$\frac{du}{dx} = 20x^3 - 6x$$

$$y = e^u$$

$$\frac{dy}{du} = e^u$$

$$\frac{dy}{dx} = \frac{\cancel{du}}{dx}\cdot\frac{dy}{\cancel{du}} = (20x^3 - 6x)(e^{5x^4 - 3x^2})$$
